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A Stochastic Model for Open Birth Interval to Estimate Primary, Adolescent and Secondary Sterilities

Introduction

THE concept of open birth interval is well-known in the renewal theory in the form of backward recurrence time of a unit at a specified time (Cox, 1962). The analysis of open birth interval has been greatly emphasised in the study of human reproduction by Srinivasan (1966, 1967 and 1968). He proposed it as a summary measure of current fertility of married women, and also suggested a set of analytical models for studying open birth intervals. Later on Leridon (1969) suggested some modifications in his results for mean and variance of open birth interval in terms of mean and variance of the closed birth interval. It has also been observed that open birth interval is quite insensitive to the change's in fecundability at least for the higher parities (Venkata-charya, 1969 and Pathak, 1971). For a fecund women, the open birth interval fails to reveal the exact fertility level if the point of the last live-birth is very near to the survey point (observation period) because its sensitivity is confounded with the truncation effect. It is, however, expected that in case of intervening secondary sterility, open birth interval may change significantly and may therefore be treated sensitive to the incidence of secondary sterility. Thus estimation of sterility is possible through the application of a probability model for the open birth interval as the secondary sterility elongates the length of the open birth interval. Pathak (1974) and Roy (1975) proposed different models to estimate the secondary sterility from the open birth interval for women of fixed marital duration. These models are derived on the assumption that the parameters of reproduction are constant throughout the period of observation. However, it is well-known that some of the parameters such as fecundability and sterility depend on parity. Sheps *et al.* (1970) obtained some general expressions for the distribution of open birth interval which take into account the variation in the parameters of reproduction. Pandey (1981) gave some exact distributions for open birth interval for fixed marital duration of the women taking the variation in the parameters according to parity. By introducing secondary sterility, recently, Singh *et al.* (1982) derived a model for open birth interval. But in countries like India, Pakistan, Bangladesh and other developing countries where most of the females consummate their marriage under the age of 20 years, the prevalence of adolescent sterility elongates the time of their first conception and consequently affects the timing of the subsequent conceptions. As such a probability model for open birth interval has been developed in this paper by considering certain proportion of women as sterile adolescents at the time of marriage. The model also takes into account the variation in the fecundability and secondary sterility by parity of a woman for a given duration of marriage. A woman may become sterile at any time in her reproductive life. The incidence of sterility may be dependent on her age or parity or both. It is, however, true that a woman will be exposed to more risk of becoming sterile at the time of termination of pregnancy. Therefore, it has been assumed here that the incidence of secondary sterility can occur to a woman only at the time of termination of pregnancy as a first approximation.

The Model

The model is derived on the basis of the following assumptions :

(1) The total period of marital life enjoyed by a woman is T years at the time of survey from the date of consummation of her marriage.

(2) None of the women are pregnant at $T = 0$.

(3) The cohort of women at the time of marriage consists of two homogeneous groups, first group comprises those who are biologically mature at the time of their effective marriage and hence are exposed to the risk of conception and the second, of those women who are not biologically mature at the time of their effective marriage but are exposed to the risk of ovulation. Let α and $(1 - \alpha)$ be their proportions in the cohort respectively.

(4) The time of first ovulatory menstruation for the adolescent sterile women follows an exponential distribution with density function

$$\mu e^{-\mu t} \qquad t > 0, \mu > 0$$

(5) The time interval of first conception after marriage follows an exponential distribution with density function

$$f_0(t) = \lambda_0 e^{-\lambda_0 t} \qquad t > 0, \lambda_0 > 0$$

(6) The duration between ith and $(i+1)$th conception follows a displaced exponential distribution with density function

$$f_i(t) = \lambda_i e^{-\lambda_i (t-h)} \qquad t > h, \lambda_i > 0 \ \forall i$$

(where h is the period of non-susceptibility including the gestation period and period of PPA).

(7) Every conception results in a live birth.

(8) Let $(1-a_0)$ be the probability that the female is primarily sterile and $(1-a_i)$ be the probability that the female becomes secondarily sterile following the termination of the ith pregnancy.

Let X_i denote the length of the open birth interval after ith birth for a female with marital duration T. Then under the above assumptions and assuming all λ_i's are distinct, the density function of X_i is given by

$$O_i(x/T) = \alpha\left[\left(\prod_{s=0}^{i-1} a_s\right) g_{2i}(x)\right] + (1-\alpha)\left[\left(\prod_{s=0}^{i-1} a_s\right) g^*_{2i}(x)\right]$$

for $O \leqslant x \leqslant h$

$$O_i(x/T) = \alpha\left[\left(\prod_{s=0}^{i} a_s\right) g_{1i}(x) + \left(\prod_{s=0}^{i-1} a_s\right)(1-a_i)\, g_{2i}(x)\right]$$
$$+ (1-\alpha)\left[\left(\prod_{s=0}^{i} a_s\right) g^*_{1i}(x) + \left(\prod_{s=0}^{i-1} a_s\right)(1-a_i)\, g^*_{2i}(x)\right]$$

for $h < x \leqslant T - g - (i-1)h$.

where

$$g_{1i}(x) = \frac{f_i(T-x)\cdot Q_i(x)}{P_i(T)}$$

$$g_{2i}(x) = \frac{f_i(T-x)}{P_i(T)}$$

$$g^*_{1i}(x) = \frac{f^*_i(T-x)\cdot Q_i(x)}{P^*_i(T)}$$

$$g^*_{2i}(x) = \frac{f^*_i(T-x)}{P^*_i(T)}$$

$f_i(T)$ is the density function of the waiting time from marriage to the ith birth, and

$F_i(T)$ is the corresponding distribution function if the woman is *not* adolescent sterile.

$P_i(T)$: Probability of i births in $(0, T)$.

$Q_i(x)$: $\int_x^{\infty} f_i(t)\, dt = e^{-\lambda_i(x-h)}$ is the probability that there is no birth in an interval of length 'x' after ith birth.

Similarly $f_i^*(T)$, $F_i^*(T)$ and $P_i^*(T)$ denote the corresponding functions when the woman is adolescent sterile at the beginning of the marriage.

Under the assumptions mentioned earlier we get,

$$f_i(T - x) = \sum_{j=0}^{i-1} \left[\frac{\left\{ \prod_{k=0}^{i-1} \lambda_k \right\} \{ e^{-\lambda_j(T-x-g-(i-1)h)} \}}{\prod_{\substack{k=0 \\ \neq j}}^{i-1} (\lambda_k - \lambda_j)} \right]$$

$$F_i(T) = \sum_{j=0}^{i-1} \left[\frac{\left\{ \prod_{\substack{k=0 \\ \neq j}}^{i-1} \lambda_k \right\} \{1 - e^{-\lambda_j(T-g-\overline{i-1}h)}\}}{\prod_{\substack{k=0 \\ \neq j}}^{i-1} (\lambda_k - \lambda_j)} \right] F_0(T) = 1$$

$$P_i(T) = \left(\prod_{s=0}^{i-1} a_s \right) F_i(T) - \left(\prod_{s=0}^{i} a_s \right) F_{i+1}(T)$$

$$f_i^*(T - x) = (-1)^i \mu \sum \left[\frac{\left\{ \prod_{k=0}^{i-1} \lambda_k \right\} \{e^{-\lambda_j(T-g-x-\overline{i-1}h)}\}}{[\lambda_i - \mu] \left\{ \prod_{\substack{k=0 \\ \neq j}}^{i-1} (\lambda_j - \lambda_k) \right\}} \right]$$

$$+\mu \left[\prod_{k=0}^{i-1} \frac{\lambda_k}{(\lambda_k - \mu)} \right] [e^{-\mu(T-x-g-\overline{i-1}h)}]$$

$$F_i^*(T) = (-1)^i \mu \sum \left[\frac{\left\{ \prod_{\substack{k=0 \\ \neq j}}^{i-1} \lambda_k \right\} \{1 - e^{-\lambda_j(T-g-\overline{i-1}h)}\}}{\{\lambda_i - \mu\} \left\{ \prod_{\substack{k=0 \\ \neq j}}^{i-1} (\lambda_j - \lambda_k) \right\}} \right]$$

$$+ \left[\prod_{k=0}^{i-1} \left(\frac{\lambda_k}{\lambda_k - \mu} \right) \right] [1 - e^{-\mu(T-g-\overline{i-1}h)}] \; F_0^*(T) = 1$$

$$P_i^*(T) = \left(\prod_{s=0}^{i-1} a_s\right) F_i(T) - \left(\prod_{s=0}^{i} a_s\right) F^*_{i+1}(T)$$

If we put $\alpha = 1$, the model reduces to that of Singh *et al.* (1982). If we put $\alpha = 1$, and $\lambda_i = \lambda$ for all /, the model reduces to that of Pathak (1974). The above results have been derived following Blyth (1949).

Application

For the application, the model may pose some problems as it has several parameters and the known methods of estimation might lead to complicated equations. However for specific order of open birth interval, the parameters will be α, a_i, λ_1 λ_2. . . λ_i, μ and these can be estimated through quasi minimum X^2 procedure.

In order to apply the proposed model to observed data to estimate the involved parameters, data should be available on the distribution of open birth interval for each parity for a cohort of women of fixed marital duration. If the data are available for different marital durations for each parity we can estimate the parity dependent secondary sterility. However, if only mean and standard deviation of the open birth interval are available for different parties of fixed marital-durations, the proposed model can be easily applied and the estimates of the parameters can be obtained. This may be done by the method of iteration.

1. From the above distribution we can easily derive the distribution of closed birth interval and apply it to the data for specific parity and get the estimates of α, μ and λ_i
2. Use the above estimates of α, μ, λ_i in the expression for the mean open birth interval of the same parity and get the estimate of a_i

Thus the derived model is capable of giving the estimates of proportion of adolescent, primary and secondary sterility, and fecundability by the parity of the women.

References

1. Blyth, B. R., 1949, Contribution to the statistical Theory of the Geiger Muller Counter, *Technical Report No. 8,* Statistical Laboratory, University of California, Berkeley, U.S.A.
2. Cox, D. R., 1962, *Renewal Theory,* London, Methuen.
3. Leridon.H., 1969, Some Comments on article by K. Srinivasan: A Probability model applicable to the study of inter live birth intervals and random segments of the same. *Population Studies,* 23, 101-104.
4. Pandey, A., 1981, A study of some probability models for birth intervals. *Ph.D. Thesis,* Banaras Hindu University.
5. Pathak, K. B., 1971, A stochastic model for the study of open birth interval—A Cohort Approach, *Sankhya,* 33 B, 305-314.

6. Pathak, K. B., 1974, Effect of secondary sterility on open birth interval, *Demography India,* 3, 133-144.
7. Roy, T. K., 1975, An analytical model for estimating the incidence of secondary sterility, *Demography India, 4,* 428-442.
8. Sheps, M. C. *et al.* 1970, The truncation effect in birth interval data, *Journal of the American Statistical Association,* 65,678-694.
9. Singh, S. N., Yadava, R. C. and Chakrabarty, K., 1982, A parity dependent model for open birth interval, *Sankhya,* 44 B, 212-218.
10. Srinivasan, K., 1966, The open birth interval as an index of fertility, *Journal of Family Welfare,* 13(2), 40-44.
11. Srinivasan, K., 1967, An analytical model for the study of interlive birth intervals and random segments of the same, *Population Studies,* 21, 63-70.
12. Srinivasan, K., 1968, A set of analytical models for the study of open birth intervals, *Demography, S,* 34-54. 13- Venkatacharya, K., 1969, Some recent findings on open birth intervals, *Artha Vijnana,* 11, 372-379.